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Mechanics of Unsaturated Soils – an alternative view

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Layout of presentation

- **Background & motivation**
- **A procedure to enhance any constitutive model to account for partial saturation**
  - ***For high degrees of saturation***
  - ***For low degrees of saturation***
- **Suction constant tests**
- **Degree of saturation- suction- void ratio relations**
- **Average pore size and its determination**
- **Influence of pore size distribution**
- **Concluding remarks**

Background & motivation

- **Great deal of research effort has been made in the past and continues to be made internationally**
- **Many new assumptions/hypotheses have been added in an ad-hoc manner to explain mechanics of USS**
- **It seems to have been generally accepted that to model behaviour of a USS requires:**
  - **an additional state parameter viz. suction (Basic Barcelona Model, 1998 and dozens more)**
  - **an relationship between degree of saturation or water content, void ratio and suction (Gallipoli et al., 2003, Salagar et al 2010)**

Background & motivation (contd.)

Some of these developments in this area seem to violate the Occam's principle

(After William of Occam (derived from the name of a village (Ockham) in Surrey, England, a fourteenth century logician)

The principle states:

"Entities should not be multiplied unnecessarily."

Or

"Pluralitas non est ponenda sine neccesitate".

The danger is that if you propose one, it might conflict with the ones which already exist and are well established

A constitutive model for partially saturated soils

- Partially saturated soil is a composite material consisting of three phases - soil skeleton, water & air
- We have already established the constitutive model for soil skeleton, We know the mechanical behaviour of water and air (Boyle's law).
- The relative volumetric measures of the three phases are dealt with in elementary soil mechanics

A constitutive model for partially saturated soils

(continued)

- From the above, the response of any partially saturated soil at any degree of saturation (S_r) , and suction (s) including their evolution can be derived
- We do need to take into account some basic characteristics of micro structure of pores

We don’t need any new assumptions/hypotheses relating to constitutive behaviour soil skeleton

A constitutive model for partially saturated soils

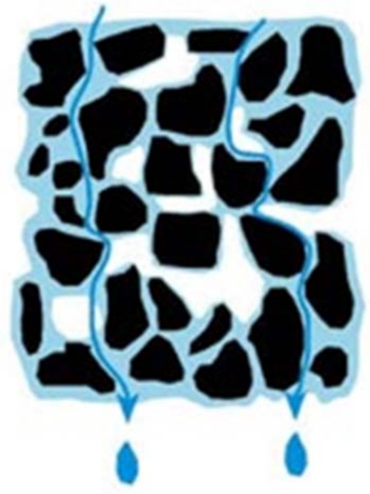
(continued)

We do need to make some assumptions relating to average pore size, pore size distribution, pore architecture and flow conditions because three different conditions may arise and transition may be discontinuous:

- Water phase continuous but air phase discontinuous
- Water and air phases both continuous
- Water phase discontinuous but air phase continuous

Partially saturated soils

at high degrees of saturation



$$\dot{\sigma}_{ij} = \dot{\sigma}'_{ij} + \dot{p}\,\delta_{ij}$$

where, *p* is the average pressure in the air-water mixture

Properties of constituents:

$$\dot{\sigma}'_{ij} = D_{ijkl}\dot{\varepsilon}_{kl}\,; \qquad \dot{p} = \bar{K}\dot{\varepsilon}^{l}_{V}$$

Where *K* is the compressibility of air-water mixture

Macroscopic constitutive relations:

$$\dot{\sigma}_{ij} = D^{*}_{ijkl}\dot{\varepsilon}_{kl}\,; \qquad D^{*}_{ijkl} = D_{ijkl} + \frac{\bar{K}}{n}\delta_{ij}\delta_{kl}\,; \qquad \dot{p} = \bar{K}\frac{\dot{\varepsilon}_{ii}}{n}$$

Partially saturated soils (contd.)

The average pore pressure in the air-water mixture, *p*, is defined as

$$p = S_r p_w + (1 - S_r) p_a - \frac{\sqrt{1 - S_r}}{\rho_v} T$$

where S_r is the degree saturation, p_w and p_a are the excess of water/air pressure respectively.
ρ_v is the **'average of pore size'** defined in a manner similar to 'hydraulic radius' in fluid mechanics, as

$$\rho_v = \frac{V_v}{S_s} = \frac{e}{S_s(1+e)}$$

where S_s is the internal solid surface area per unit volume and e is the void ratio.

Partially saturated soils (contd.)

$$\dot{p}_w = K_f \dot{\varepsilon}_{ii}^{w}; \quad \dot{p}_a = K_a \dot{\varepsilon}_{ii}^{a}; \quad K_a = p_a + p_{a0}$$

Boyle's law

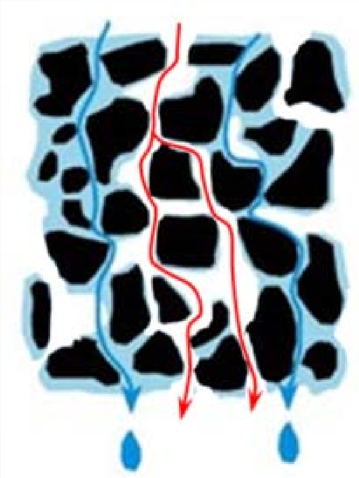
$$\dot{\varepsilon}_{ii}^{a} = B_a \dot{\varepsilon}_{ii}; \quad \dot{\varepsilon}_{ii}^{w} = B_w \dot{\varepsilon}_{ii}$$

$$B_w = \frac{1}{n\left[S_r + (1-S_r)\dfrac{K_f}{K_a - \beta_h}\right]}; \quad B_a = \frac{K_f}{K_a - \beta_h} B_w$$

$$\beta_h = \frac{T}{3\rho_V} \frac{S_r}{\sqrt{1-S_r}}$$

$$\frac{\bar{K}}{n} = B_w K_f \Rightarrow \bar{K} = \frac{K_f}{S_r + (1-S_r)\dfrac{K_f}{K_a - \beta_h}}$$

USS at low degrees of saturation



Water phase and air phase both are continuous

Following some mathematics of composite materials, the eqn. for pressure of water/air phase can be reduced to

$$p = S_r p_w + (1 - S_r) p_a - \frac{T}{\rho_v}$$

; where *T* is surface tension and ρ_v

$$\rho_v = \frac{n}{S_m}$$

where *S* is the surface area of water menisci which is less than S_s

ρ_v is again an average or 'characteristic' pore diameter as defined before

USS at low degrees of saturation (contd.)

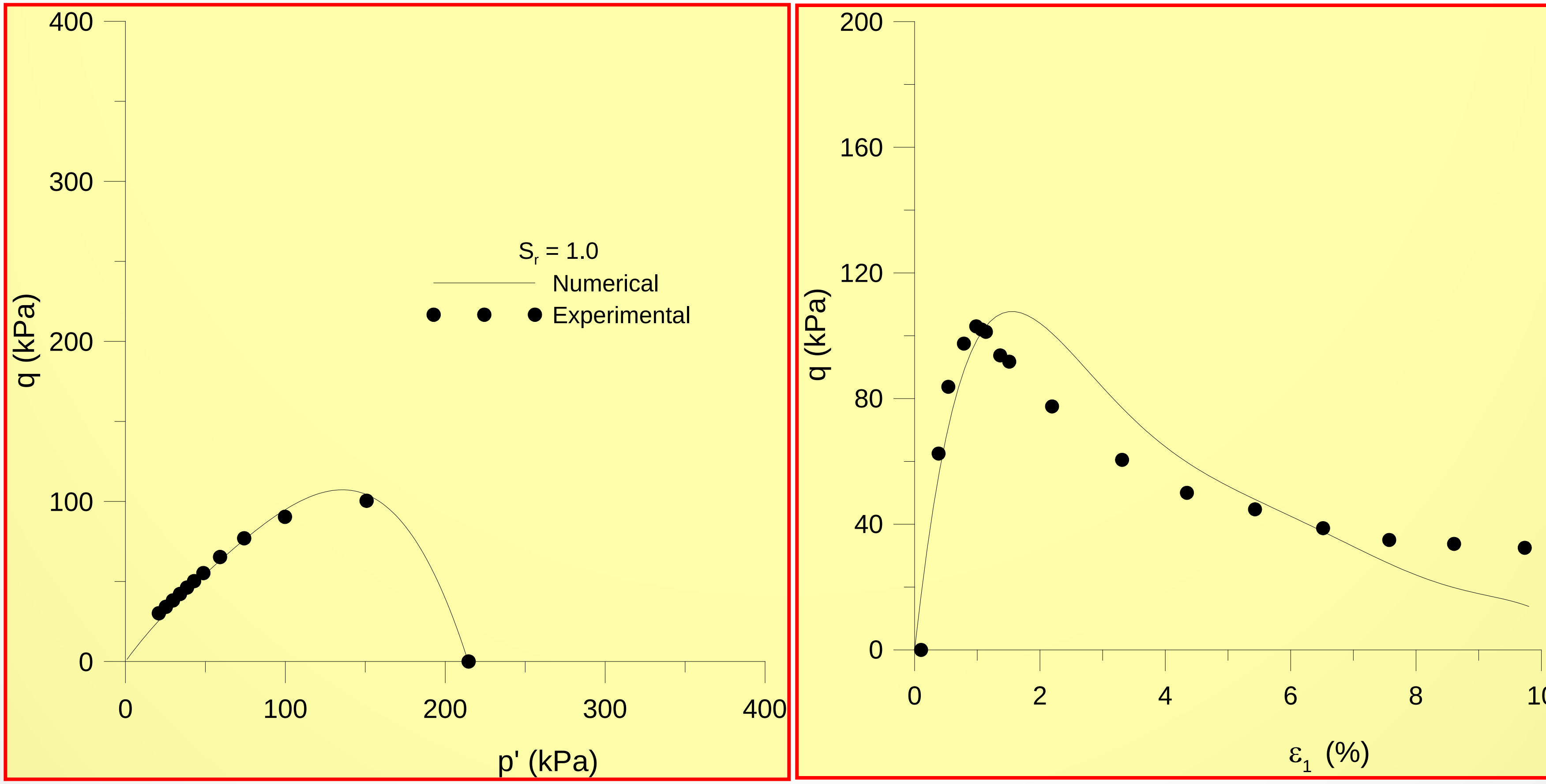
In this case, suction (s) is given by:

$$s = p_a - p_w = \frac{T}{(1 - S_r)\rho_v}$$

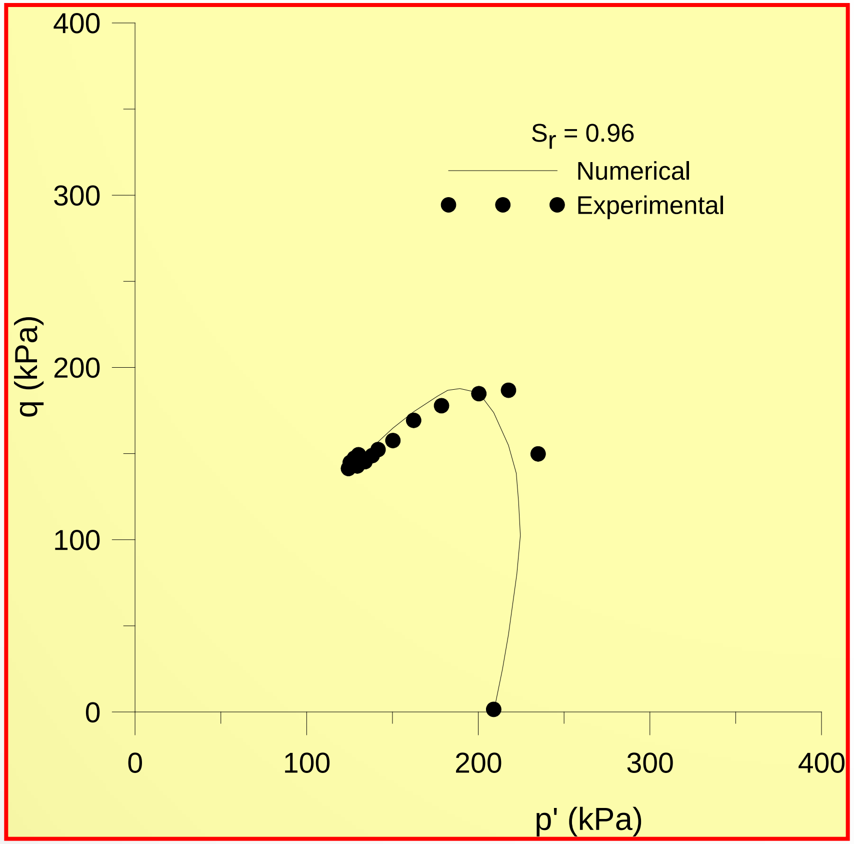
Refer to "On the mechanics of partially saturated soils", Computers &Geotechnics, 1991
Also ASCE Geotech Eng. censored version, 1993?
"On the mechanical response of partially saturated soils at low and high degree of saturation",

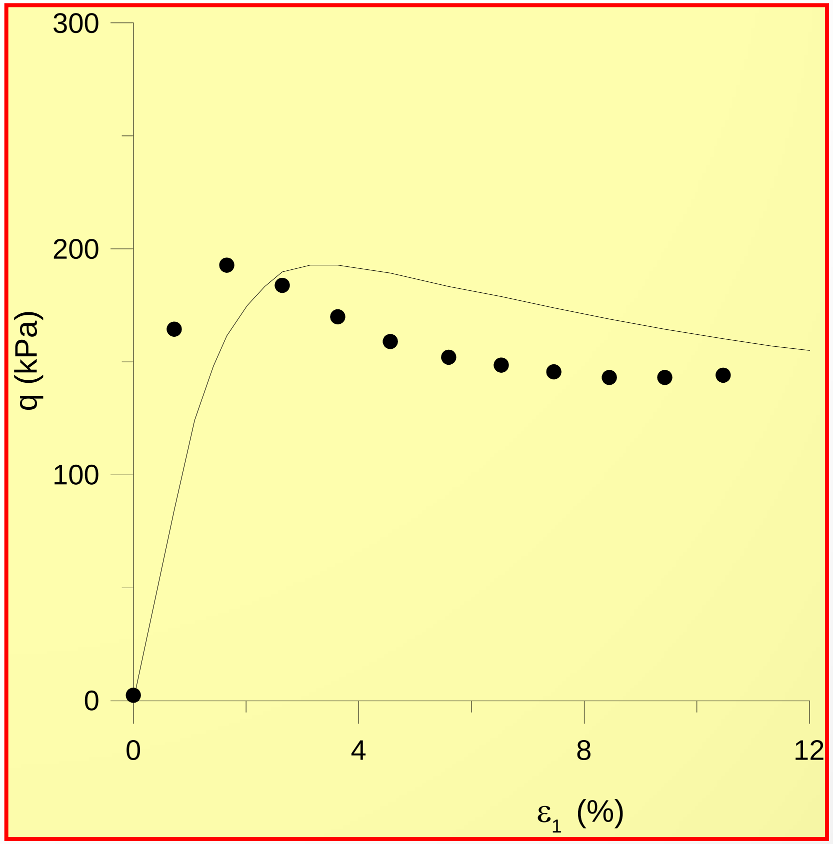
Proc. Num. Models. Geomech. NUMOG V, Davos, Balkema, 1995

USS at low degrees of saturation (contd.)

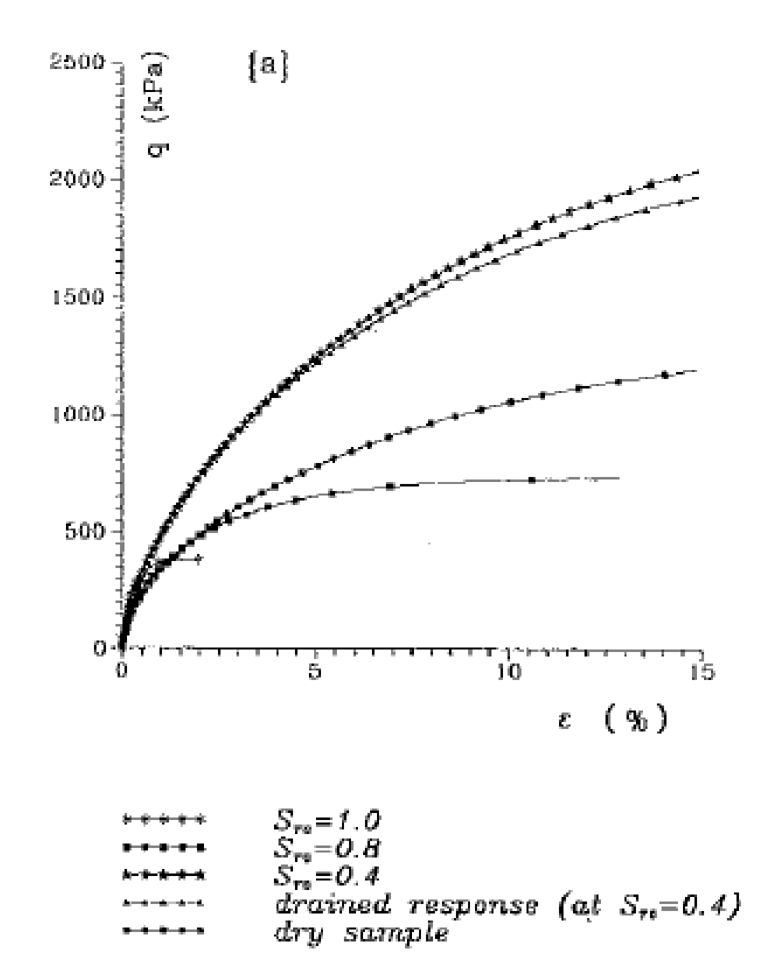


USS at low degrees of saturation (contd.)





USS at low degrees of saturation (contd.)



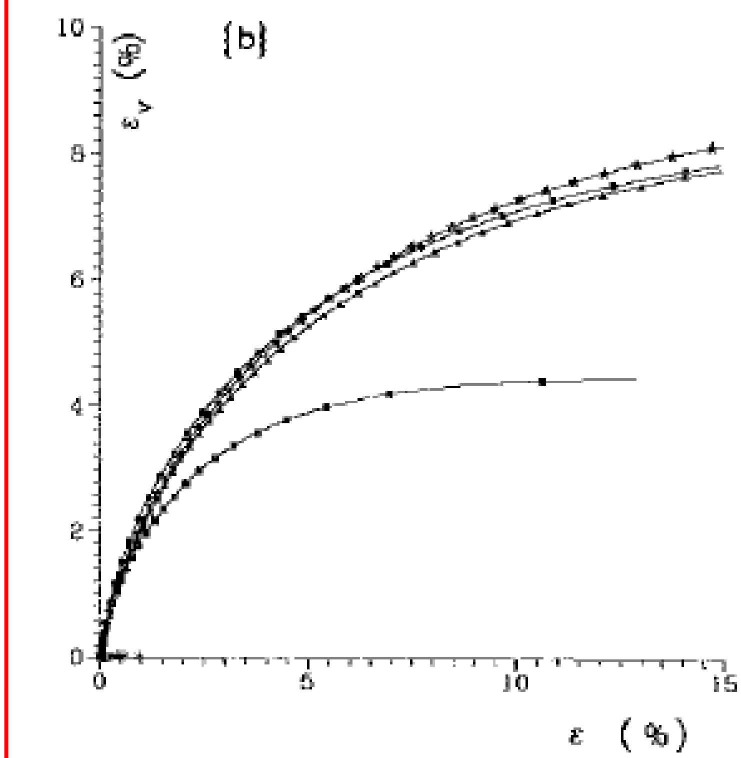
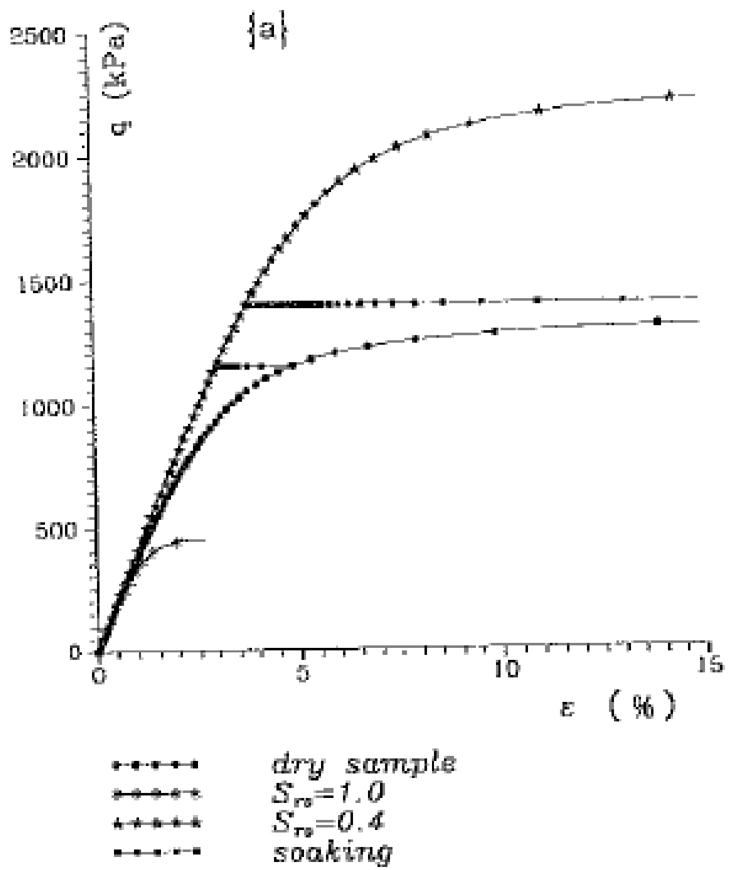


Fig.1. Numerical simulations of uniaxial compression tests on unstaurated normally consolidated clay;
(a) deviatoric characteristics ($q=\sigma_1-\sigma_3$; $\varepsilon=2(\varepsilon_1-\varepsilon_3)/3$)
(b) volume change characteristics ($\varepsilon_v=\varepsilon_1+2\varepsilon_3$)

USS at low degrees of saturation (contd.)



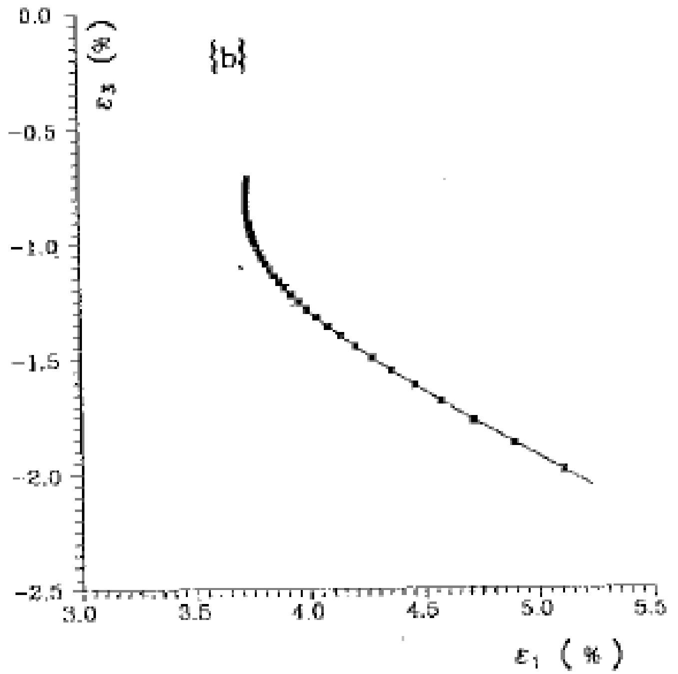
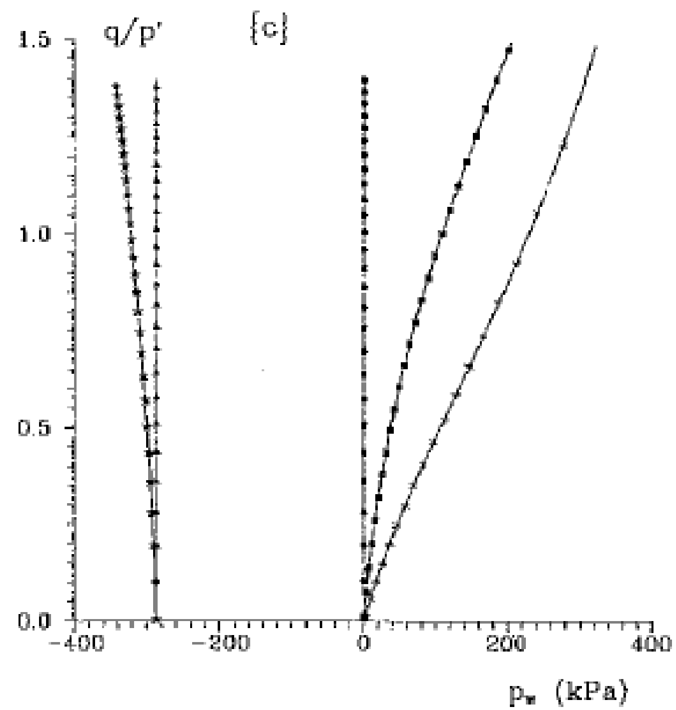


Fig.2. Uniaxial compression of unsaturated clay followed by soaking; predictions based on deviatoric hardening model; (a) deviatoric characteristics, (b) evolution of principal strains due to soaking (at q = 1150 kPa).



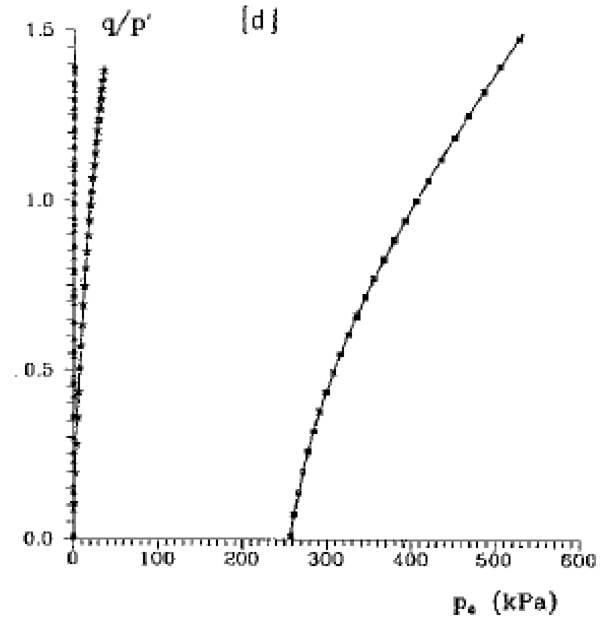


Fig. 1. cont.
(c) and (d): evolution of the excess of pore water and air pressure, respectively.

Constant suction tests

- Tests under ‘constant suction’ are essentially drained tests and as such demonstrate the behaviour of soil skeleton and give no additional information

- Unfortunately, a large number of tests reported in literature are of this type

- Can some one do a series of undrained tests?

Constant suction tests

Test #	p (kPa)	S (kPa)	q_f red (kPa)	q_f calc (kPa)	% Error
1	142	9.7	132	131	0
2	142	14.2	149	135	9
3	137	95	228	200	12
4	280	16	241	255	6
5	340	0	293	293	0
6	475	10	370 [b]	417	13
7	390	95	377	418	10
8	325	91	384	360	6
9	544	40	500	503	1
10	557	41	500	515	3
11	558	16	508	495	3
12	590	97	567	590	4
13	540	95	567	549	3

This value appears to be too low since even without suction the strength should have been higher than 410 kPa in view of test # 5.

Degree of saturation, suction, void ratio relationship

Many researchers e.g. Gallipoli et al (2003), Salager et al (2010) have investigated s-e-Sr or s-e-w relationships.

The former propose:

$$S_r = \left\{ \frac{1}{1 + \phi \, e^{\psi} s^n} \right\}^m$$

$$dS_r = d\left(\frac{V_w}{V_v}\right) = \frac{dV_w}{V_v} - \frac{V_w dV_v}{V_v^2} = -S_r \frac{de}{e} \qquad (1)$$

$$-\frac{de}{1+e} = d\varepsilon_V \qquad (2)$$

Substituting (2) in (1)

$$dS_r = S_r \frac{(1+e)}{e} d\varepsilon_V \qquad (3)$$

$$e = \exp^{-\varepsilon_V} - 1 \qquad (4)$$

$$dS_r = S_r \left[\frac{\exp^{-\varepsilon_V}}{\exp^{-\varepsilon_V} - 1}\right] d\varepsilon_V \qquad (5)$$

Assuming soil as elasto-plastic, represented by the critical state model

$$d\varepsilon_V = d\varepsilon_V^e + d\varepsilon_V^p \tag{6}$$

$$= \frac{\kappa}{1+e}\frac{dp'}{p'} + \frac{\lambda-\kappa}{p'(1+e)(M^2+\eta^2)}\left[(M^2-\eta^2)dp' + 2\eta dq\right]$$

where λ, κ are well known parameters of clays, η is the stress ratio = q/p')
For isotropic compression η=0 leads to:

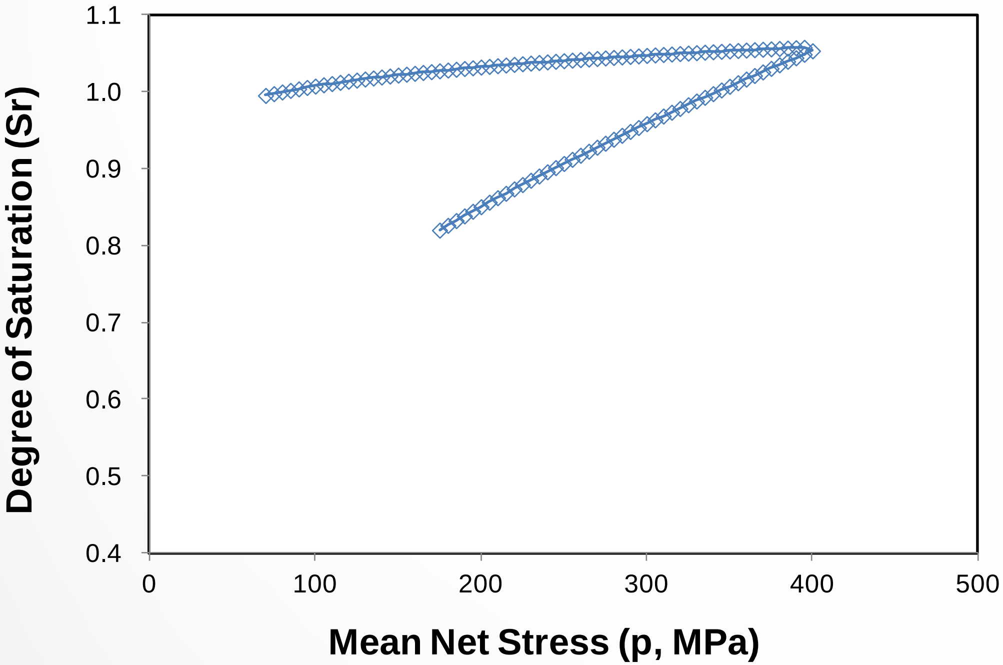
$$d\varepsilon_V = \frac{\lambda}{(1+e)}\frac{dp'}{p'} = \frac{\lambda}{e^{-\varepsilon_V}}\frac{dp'}{p'} \tag{7}$$

$$dS_r = S_r\left[\frac{\lambda}{e^{-\varepsilon_V}-1}\right]\frac{dp'}{p'} \tag{8}$$

which can be simplified to $S_r = S_{r_0}\left[\frac{p'}{p'_0}\right]^{\lambda/e}$ (9)

$$S_r = S_{r_0}\left[\frac{p'_0}{p'}\right]^{\kappa/e}$$

This can be compared with empirical equations proposed by many researchers.



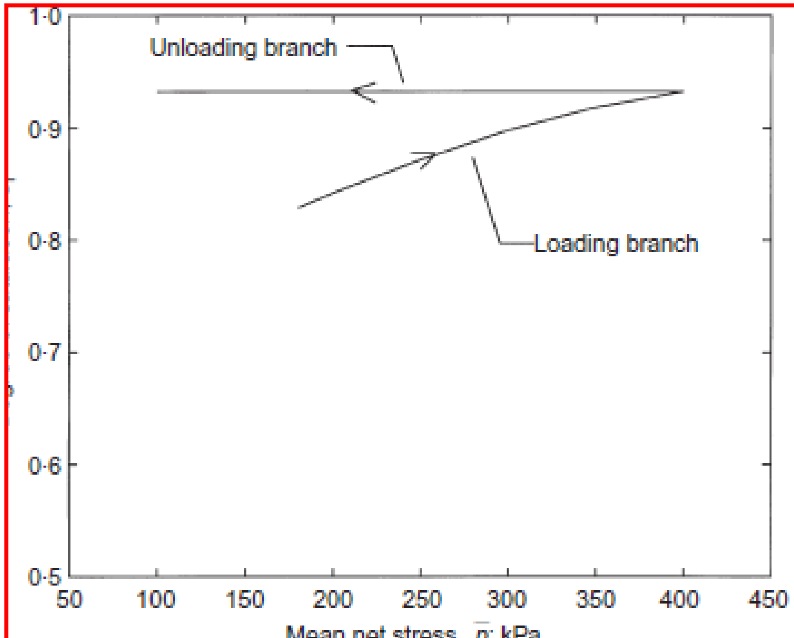
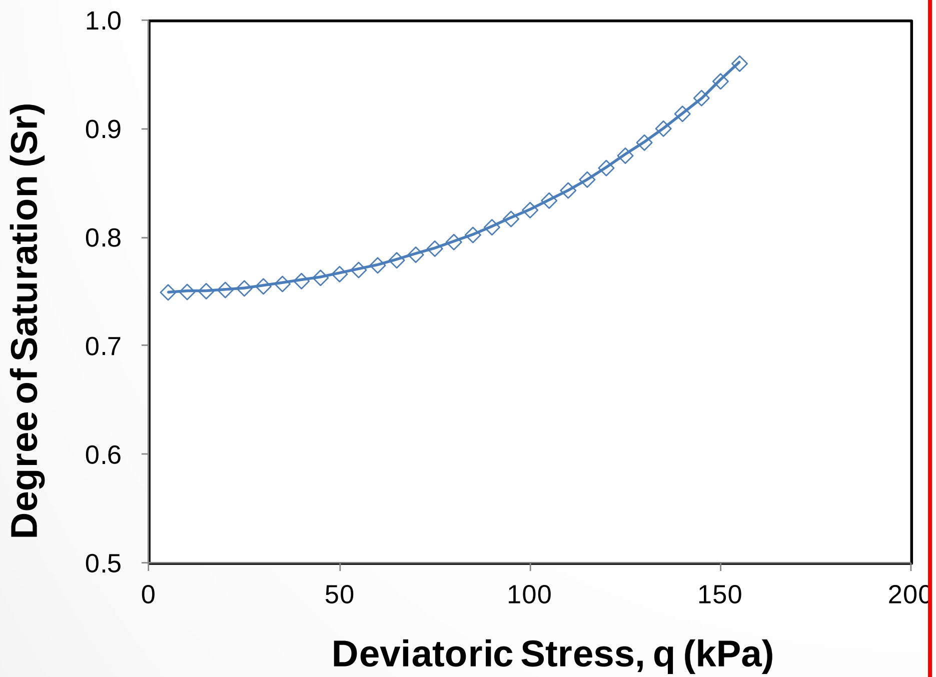


Fig. 1. Isotropic loading–unloading test at constant suction on compacted Speswhite kaolin (Zakaria, 1995)



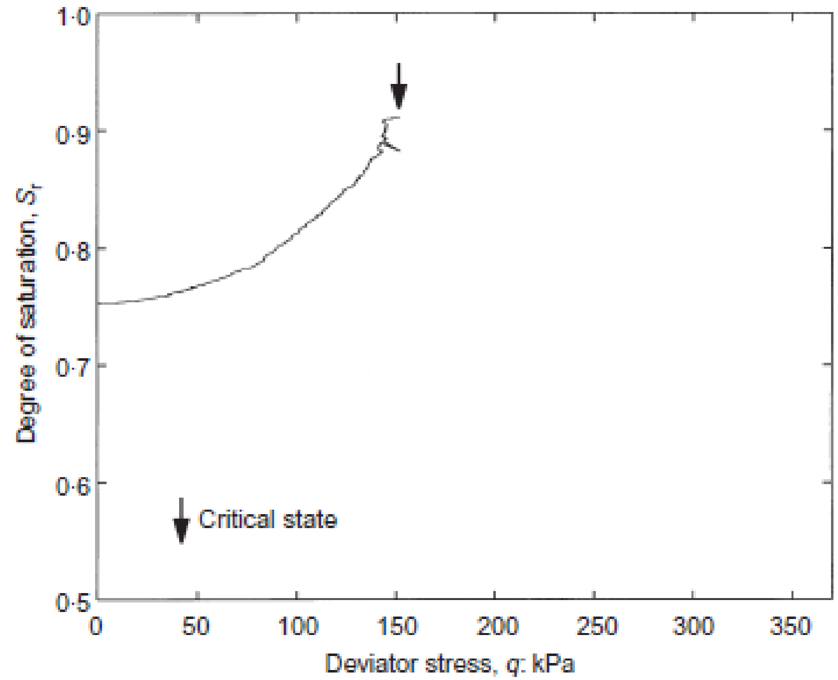
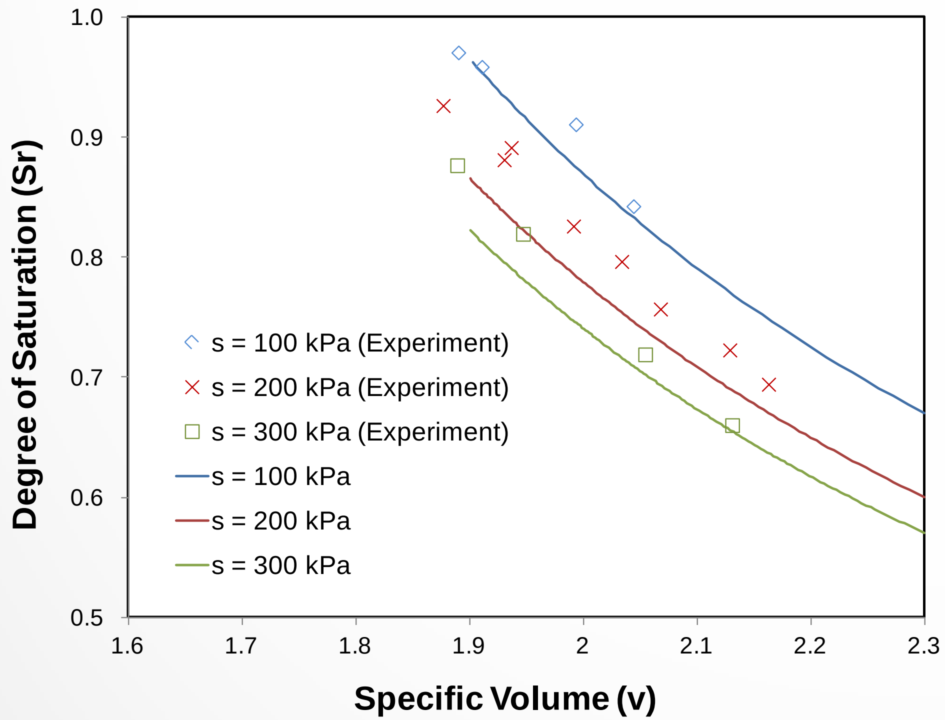


Fig. 2. Triaxial shear test at constant suction and constant mean stress on compacted Speswhite kaolin (Sivakumar, 1993)

Figure 2



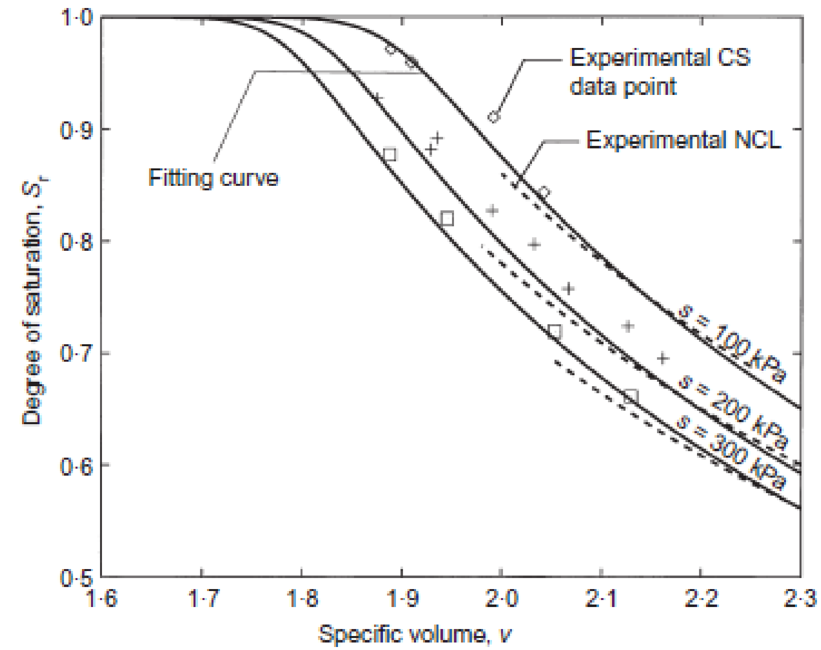


Fig. 3. Values of S_r on normal compression lines and at critical states for compacted Speswhite kaolin (experimental data from Sivakumar, 1993)

Figure 3

After Campbell 1984

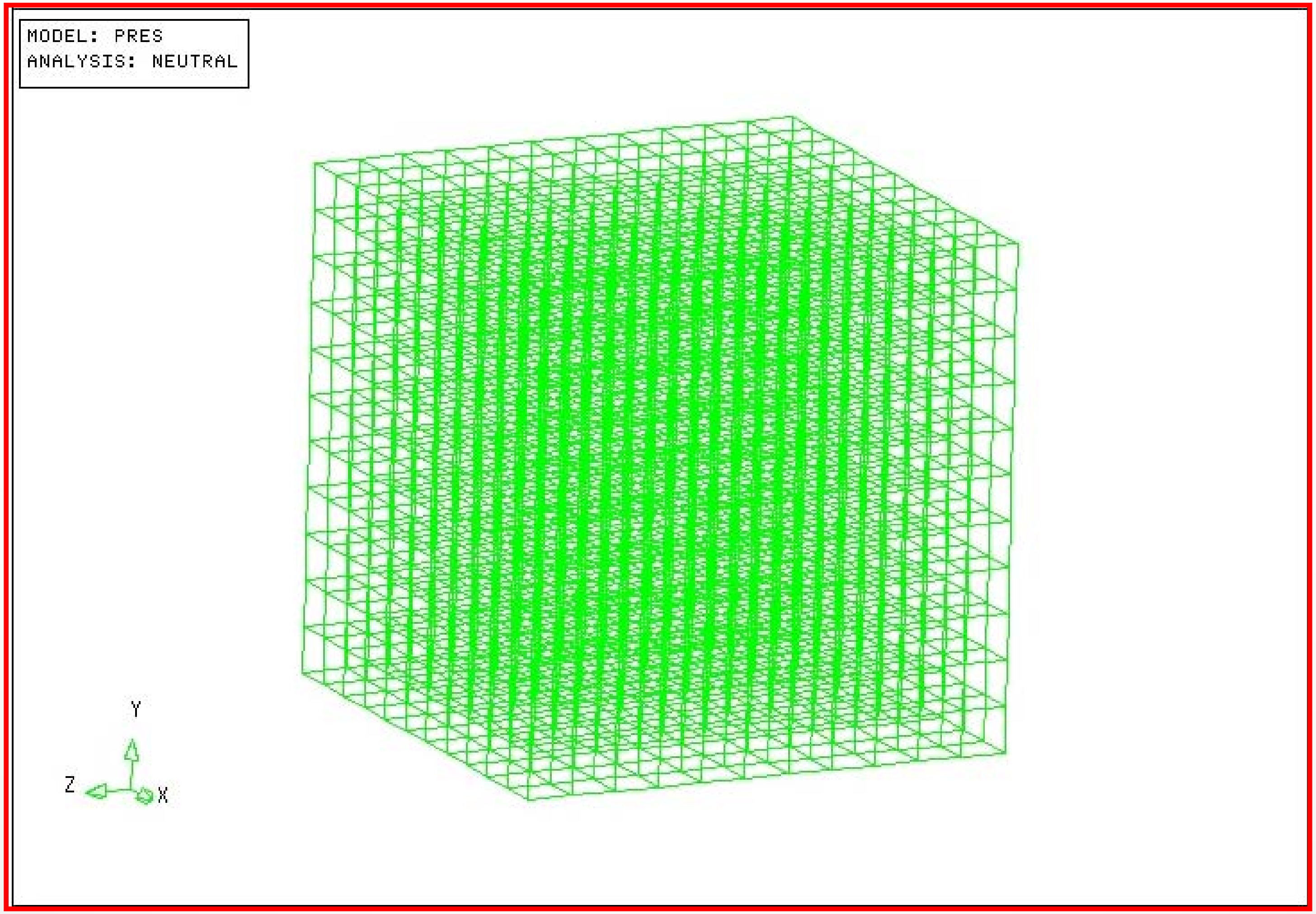
				Size				
	Coarse	Medium	Fine	Very Fine				
Sorting	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower
Ewl sorted	475.	238.	119.	59.	30.	15.	7.4	3.7
Vw sorted	458.	239.	115.	57.	29.	14.	7.2	3.6
Well sorted	302.	151.	76.	38.	19.	9.4	4.7	2.4
Moderately sorted	110.	55.	28.	14.	7.	3.5		
Poorly sorted	45.	23.	12.	6.				
Very poorly sorted	14.	7.	3.5					

Determination of ρ_v

Three possible approaches:

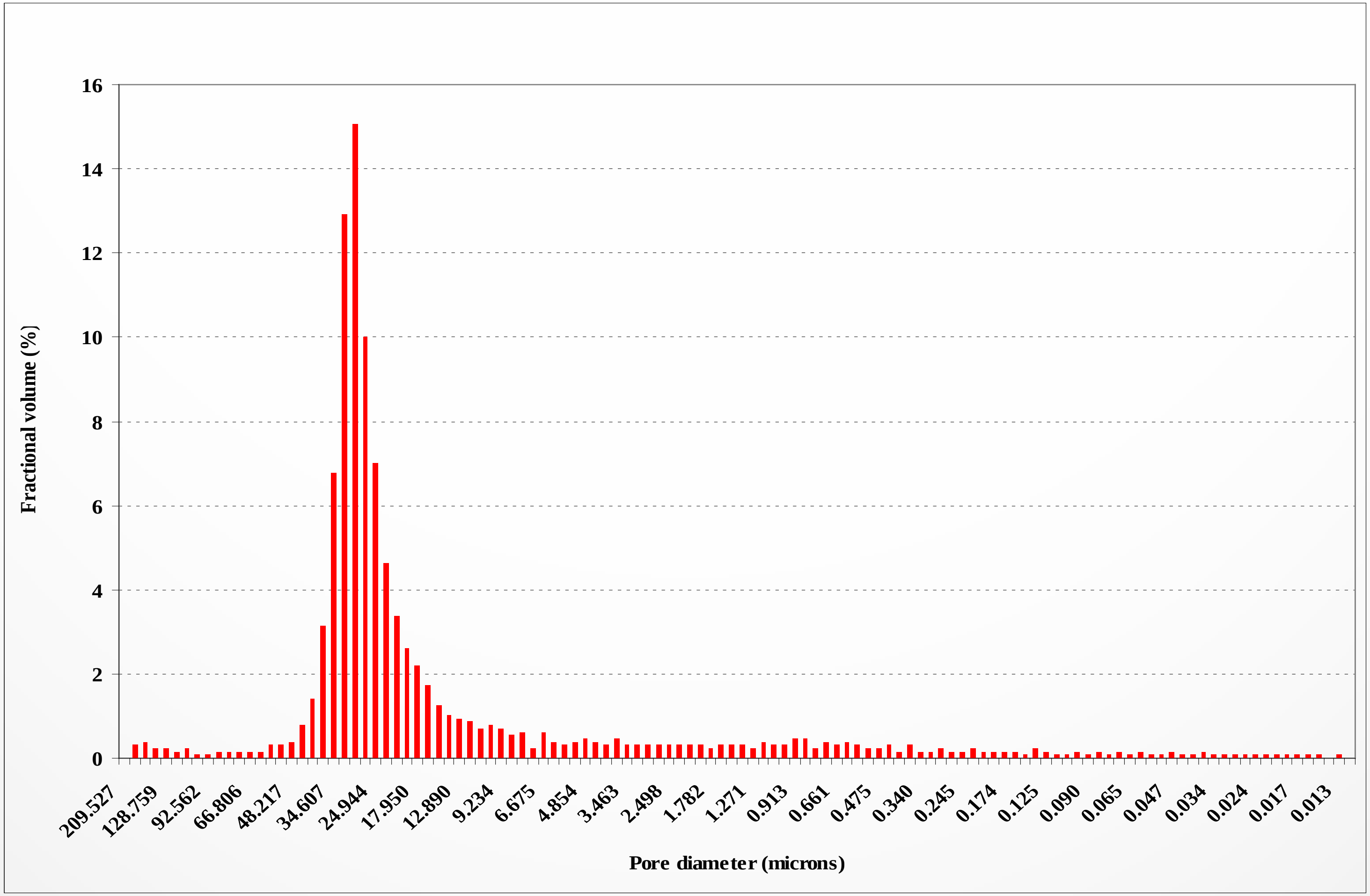
- Specific surface area (S_s)can be measured for any soil using many standard techniques. There are standard tests used in chemical & petroleum engineering and cement industry which can be adapted for soils
- Average pore size and its distribution has been correlated (Arya & Paris (1985), Imre (2008, 2012) and many others) to particle size distribution (gradation curve) assuming grain shape as
  - ✓ spherical
  - ✓ oblong ellipsoidal or platelets (can be done)
- Rapid advances have been made in new technology of X-ray computer tomography

SWRC - Pore network model



Pore diameter histogram

Test L4 by MIP



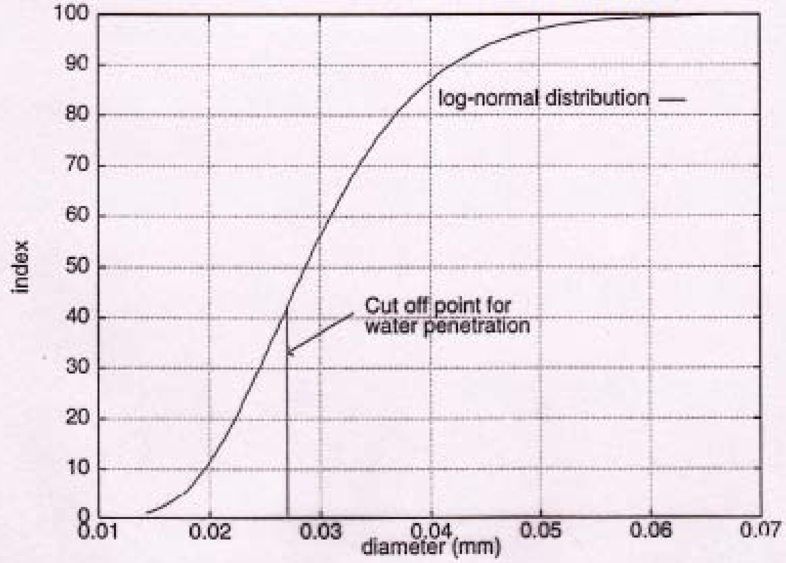
Probability distribution of pipe diameters in the network is defined by

$$f(d) = \frac{1}{\zeta d \sqrt{2\pi}} \exp \frac{-(\ln d - \lambda)^2}{2\zeta^2}$$

f(d) IS THE PROBABILITY DENSITY FUNCTION

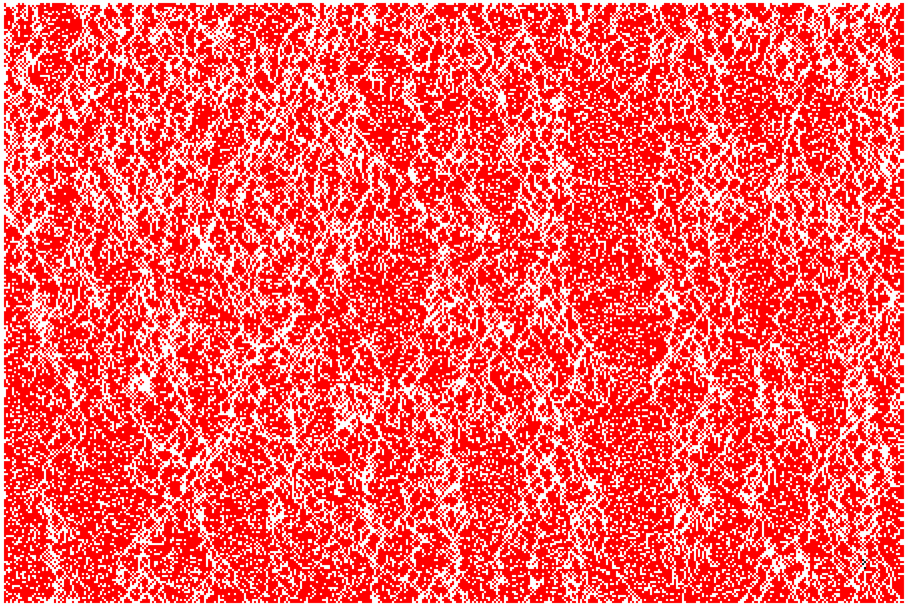
$$P_k = \int_{d_k}^{d_1} f(d)\, dD$$

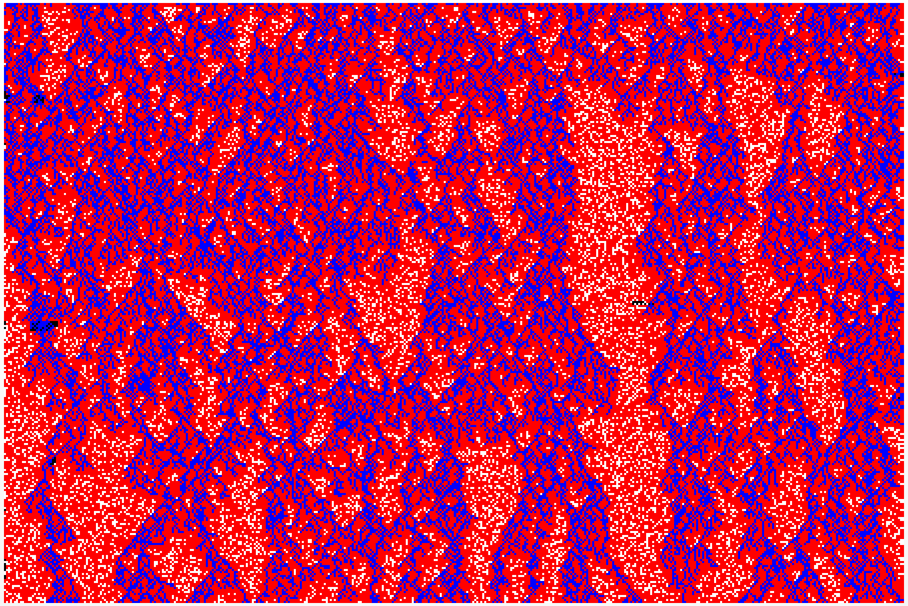
d_k IS PORE DIA
d_1 IS THE LARGEST PORE DIA
P_k IS THE CUMULATIVE INDEX FRACTION



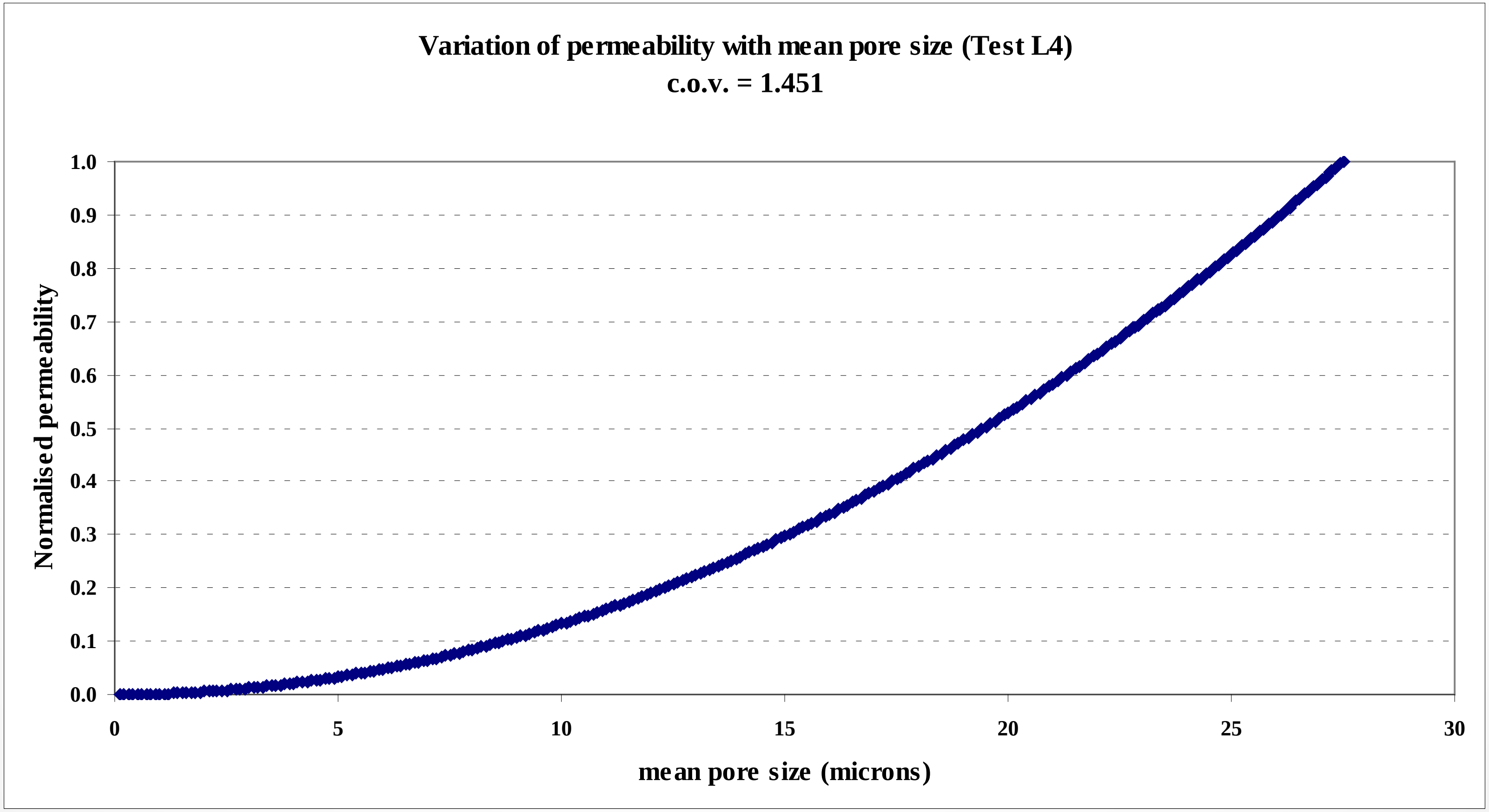
Log-normal cumulative distribution curve

Visualisation of pore network and percolation

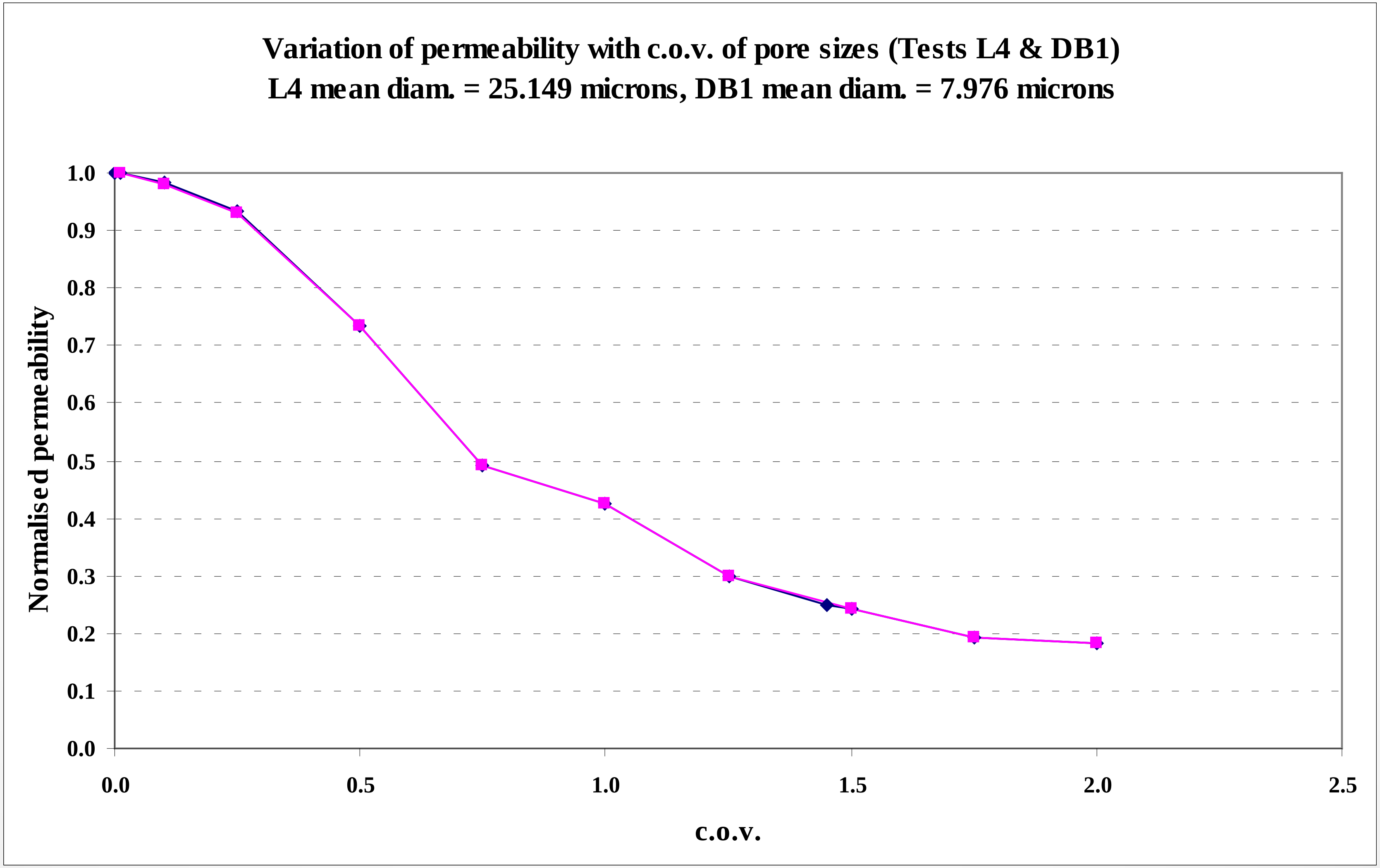




Variation of permeability with mean pore size

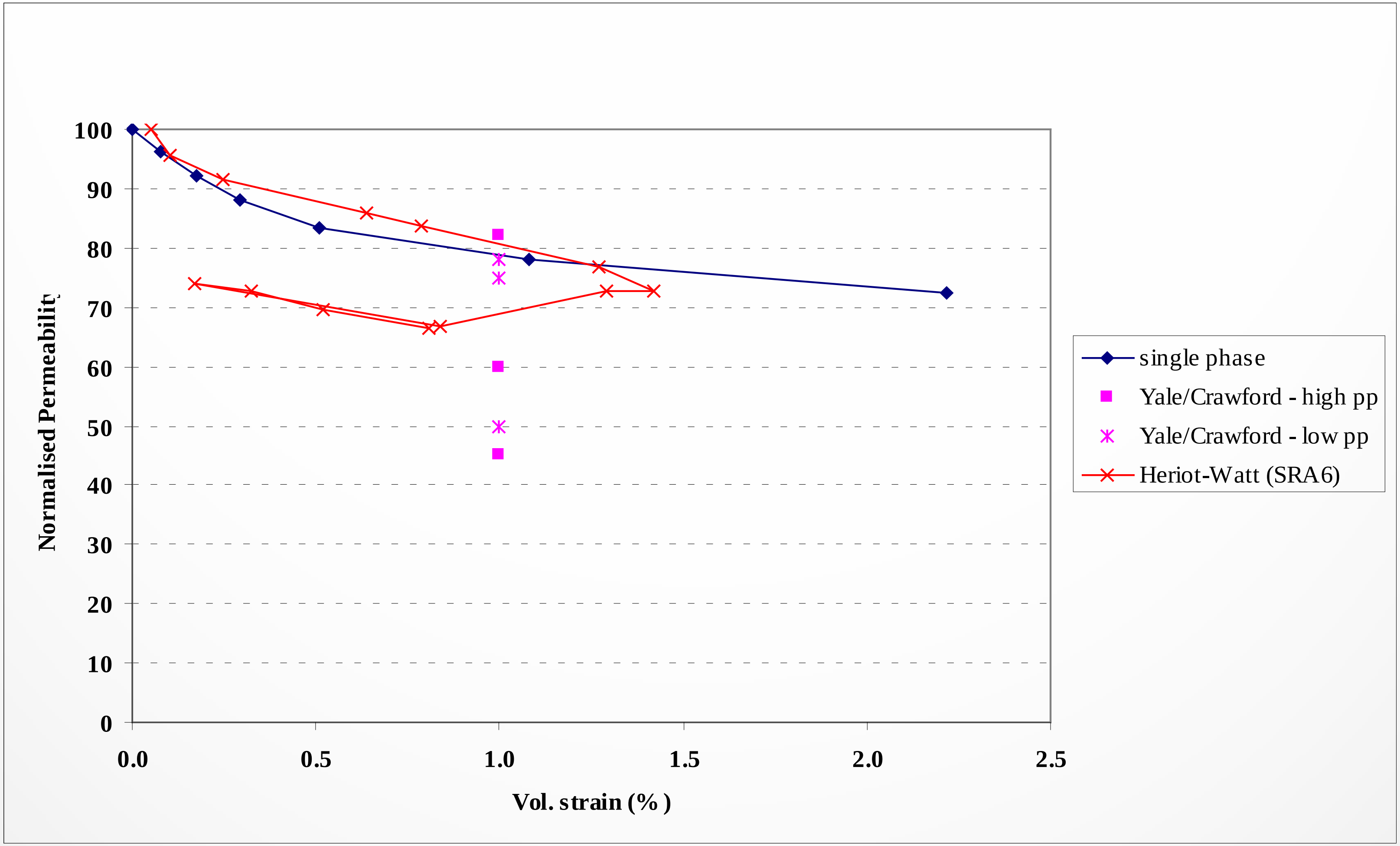


Permeability variation with C.O.V.



Permeability variation

on isotropic loading/unloading

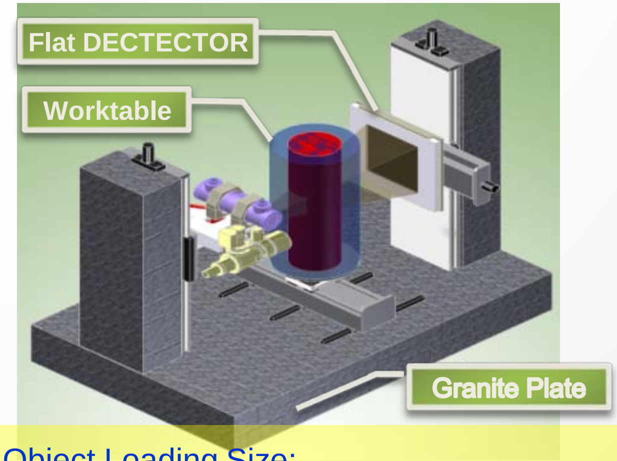


X-ray CT facility for live tests in KICT

Dimension	3000 x 1850 x 2700 (㎜)
Weight	20,000㎏

- **<u>Switchable hige-power multi tube type</u>**

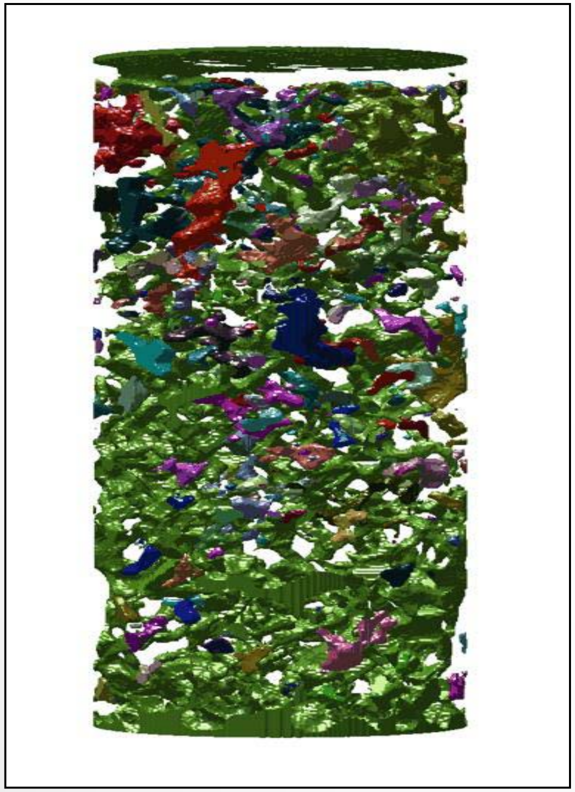
1) High power target X-ray tube (**320kV**)
 > Closed High power tube (FSS: 0.4mm)
2) Directional target X-ray tube (**225kV**)
 > Micro focus open high power tube (FSS: 6μm)
3) Transmission target X-ray Tube (**120kV**)
 > Nano Focus Open Tube (FSS: 400nm)



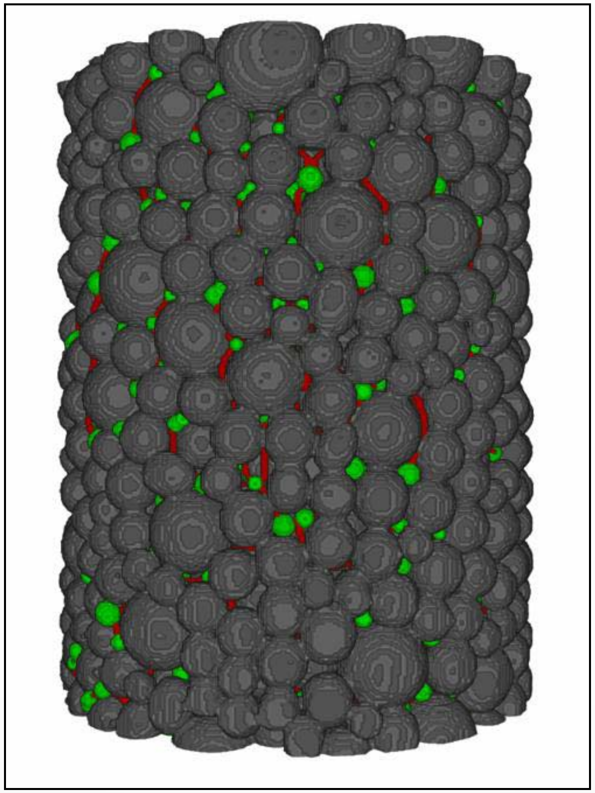
- Object Loading Size: max. ø500mm x 1000mm(h)
- Work Table Withstand load: max. 100kg
- 3DCT area: ø300mm x 900mm(h)

X-ray CT facility for live tests in KICT

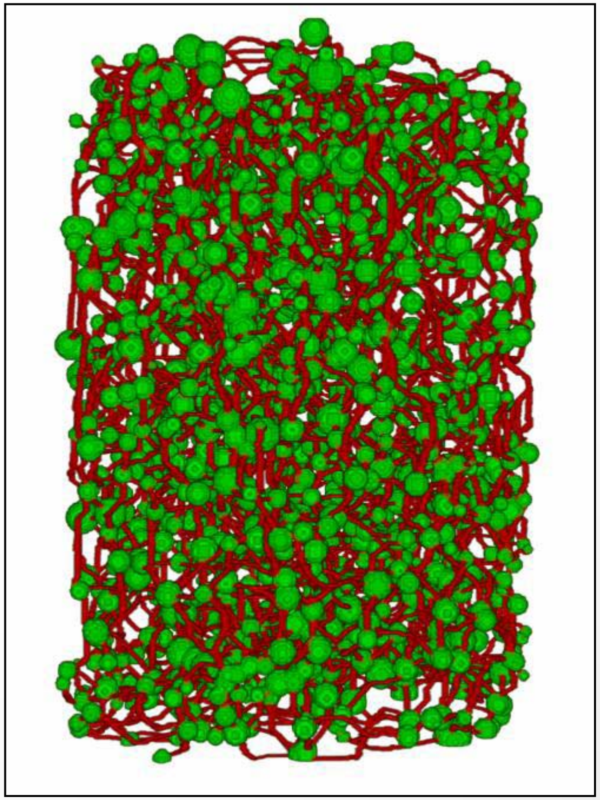
- **Identification of pores/particles/other phases**
  - **- Identification of shape and size of multi-phased distribution**
  - **- Calculation of equivalent spheres to each volume of detected shapes**
- **Determination of channels through connection of pores**



(a) Original shapes



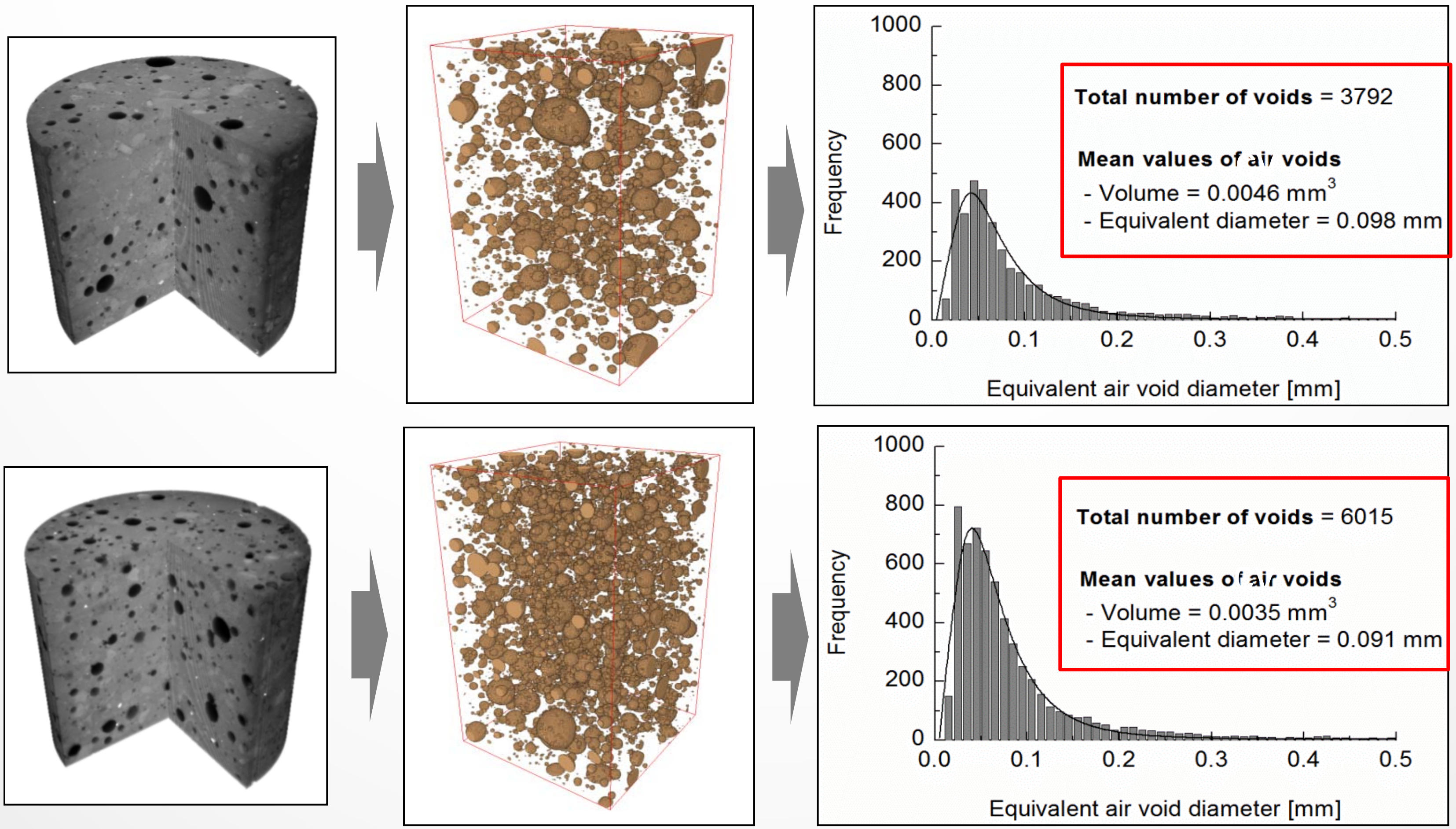
(b) Classification of material phases



(c) Identification of connectivity and channels

X-ray CT facility for live tests in KICT

- **Quantification of identities of constituents in specimen**
  - Identification of each individual constituents in specimen
  - Quantification of individual identities and statistics of whole composition



X-ray CT facility for live tests in KICT

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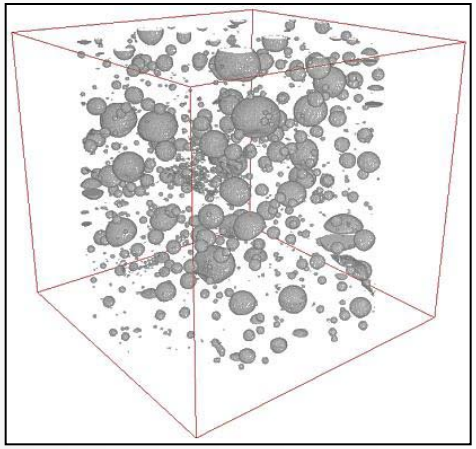

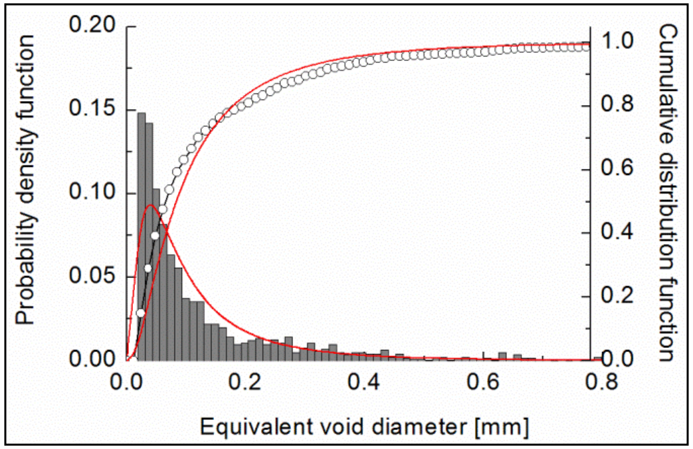

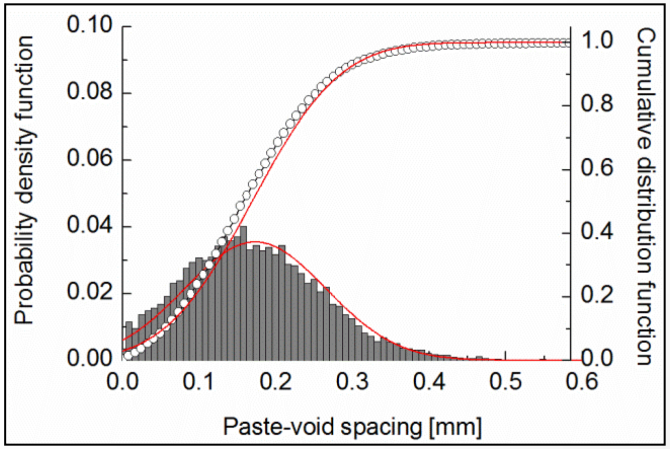

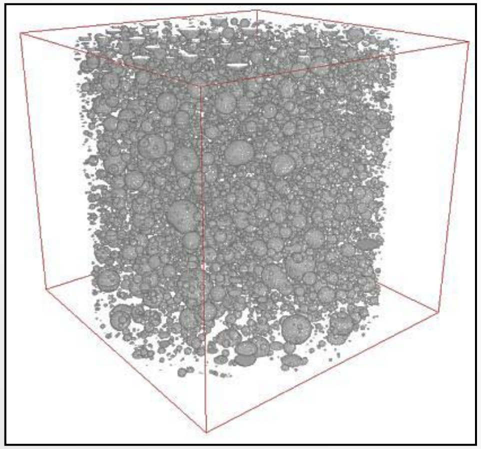

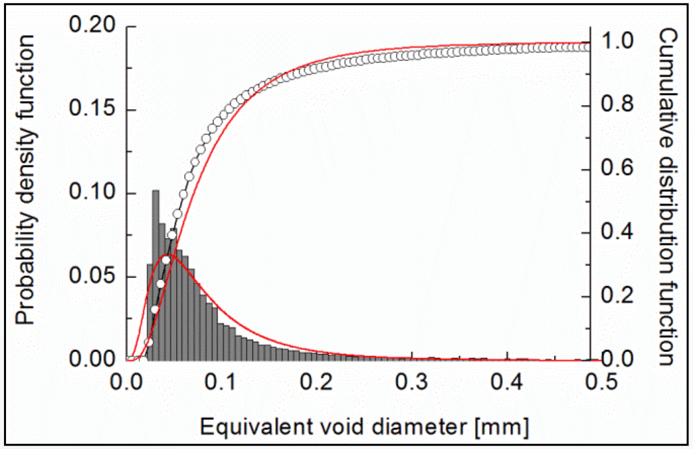

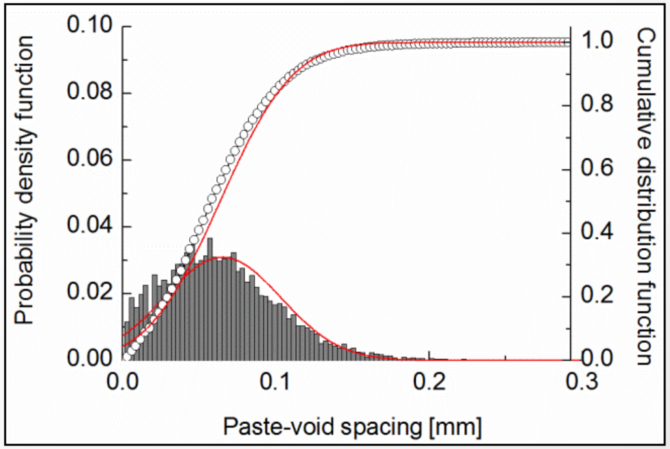

Concluding remarks

- It has been shown that a partially saturated soil can be treated as a composite material and its constitutive relation can be obtained simply by applying existing mathematical procedures of volume averaging.
- When this approach is adopted, evolution of S_{r}, p_{a} and p_{w} with stress are traced.
- The additional parameters required for characterising the mechanical response of such soils relate to microstructure as described by a characteristic 'pore size' and its 'distribution' as well as its evolution during loading.
- Particle size distribution or 'gradation curve' are fundamental characteristic of soils and are deeply embedded in engineering practice. Pore size distribution is related to gradation curve.

Concluding remarks (contd.)

- **It is important to develop a unified program of live testing with observations at the micro x-ray computer tomographic level.**
- **This will not be at the pore level for clays but interpretation at voxel level will be sufficient for practical purposes.**

KICT and IC2E would like to have ‘expression of interest’ in a collaboration programme involving CT imaging, experimental testing and computational advances.

Thank you for your attention